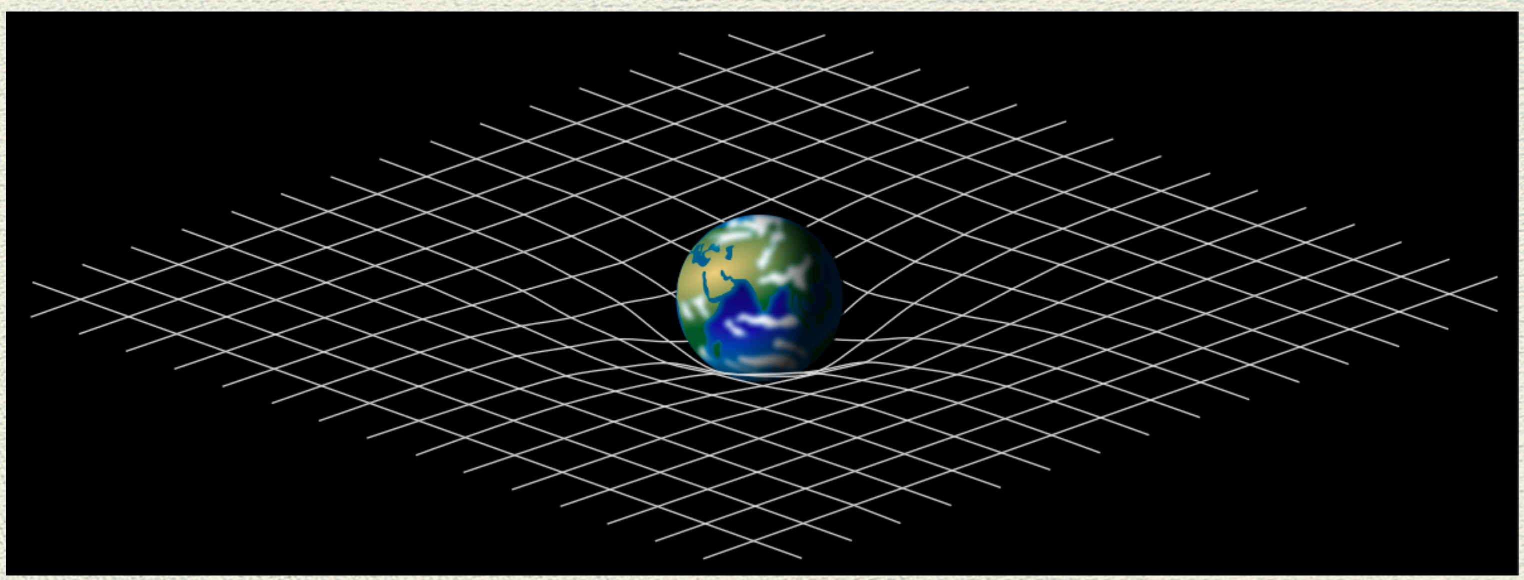
Holography and Quantum Error Correction

Part II- Anti de Sitter Space

Einstein's Equation



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

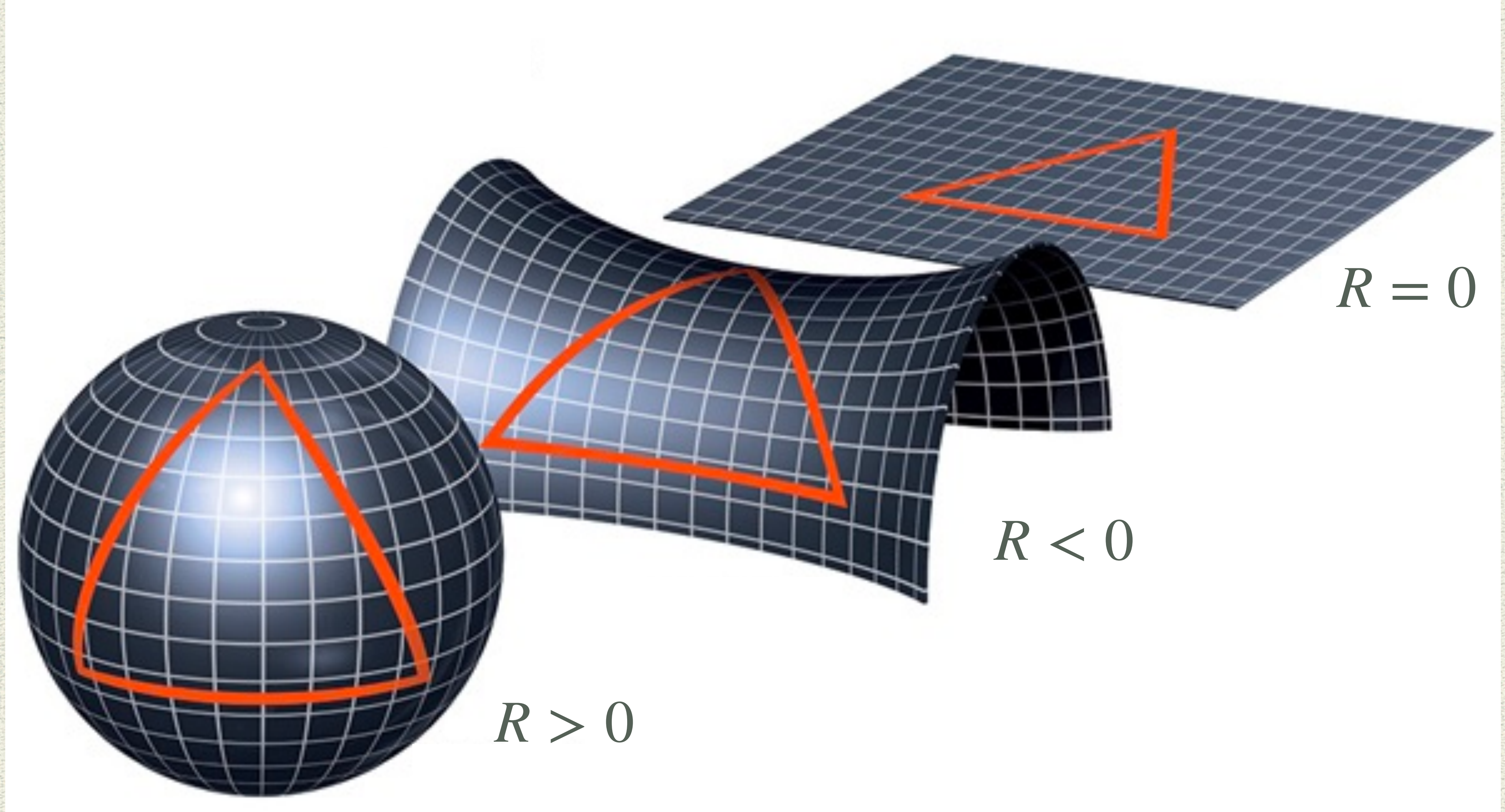
$$G_{\mu\nu} = \frac{8\pi G}{c^3} T_{\mu,\nu}$$

$$G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

Cosmological principle = homogeneity and isotropy

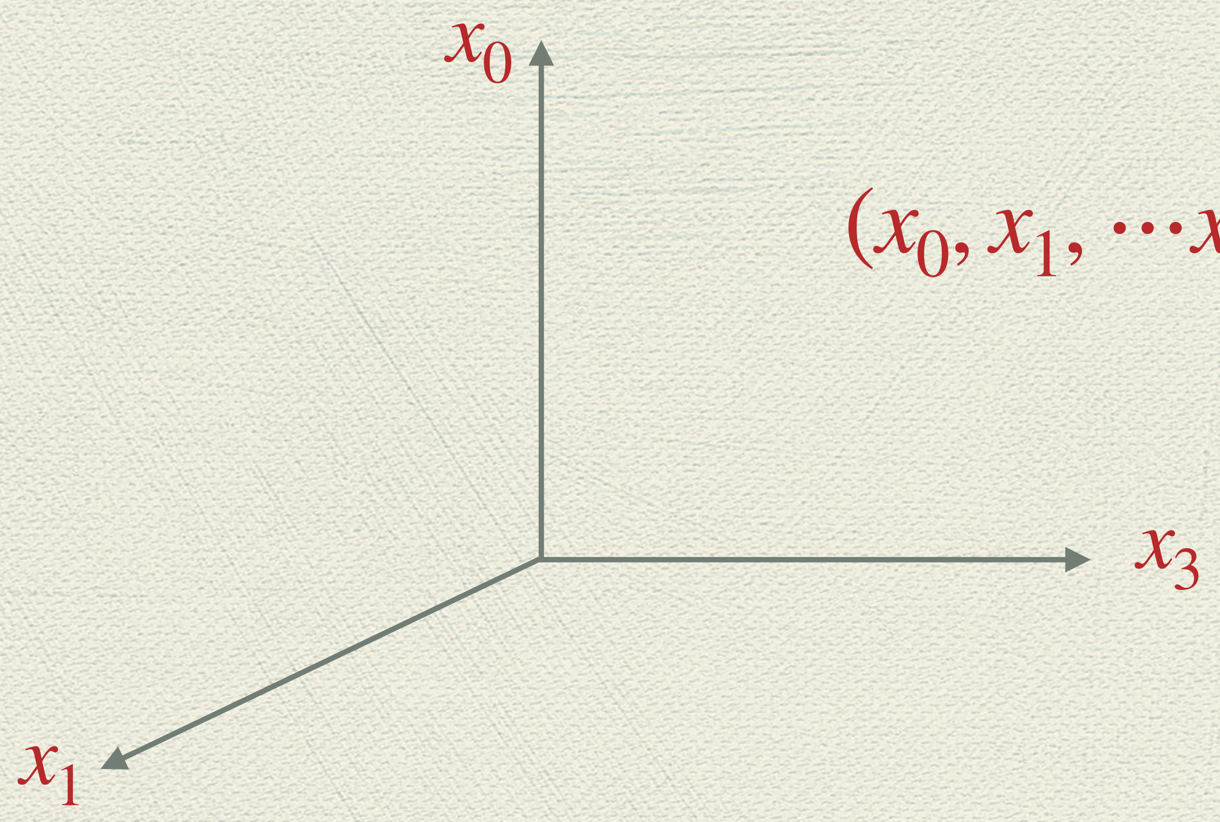
We are interested in space-times of constant curvature with Lorentzian signature

In 2 dimensions



A-Constant Zero curvature in 4-dimension

Flat spacetime



$(x_0, x_1, \cdots x_3)$

Metric of ambient space $ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - dx_0^2$

Polar coordinates

$$x_0 = t$$

$$x_1 = r \sin\theta \cos\phi$$

$$x_2 = r \sin\theta \sin\phi$$

$$x_3 = r \cos\theta$$

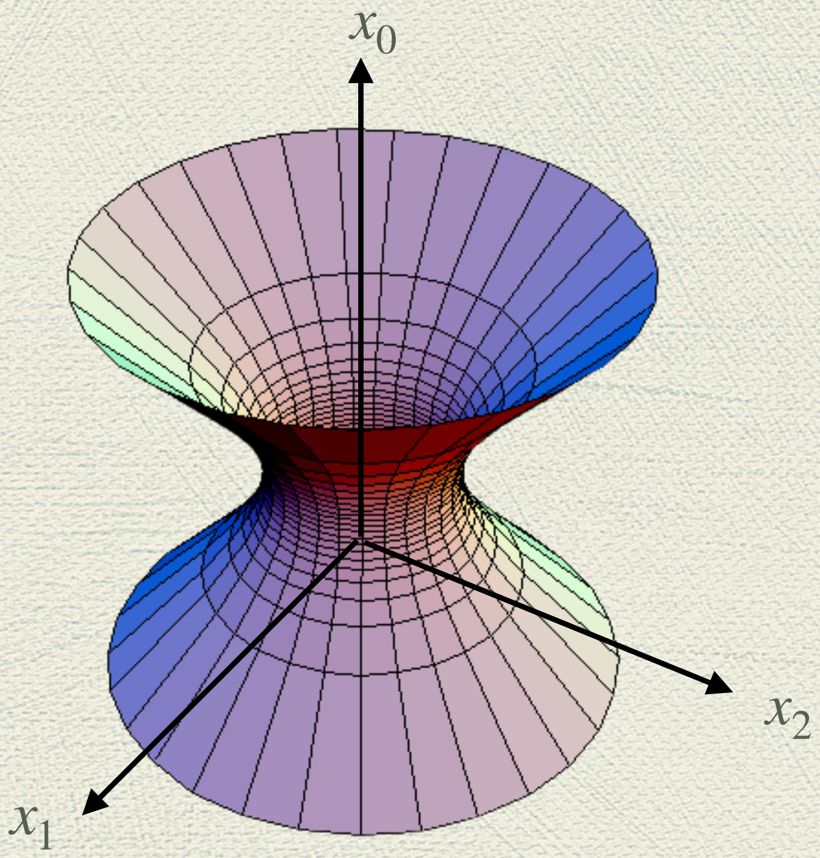
$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - dx_0^2$$

↓

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 - dt^2$$

B-Constant Positive curvature in 4-dimension

de Sitter Spacetime dS^4



$$x_1^2 + x_2^2 + x_3^2 + x_4^2 - x_0^2 = a^2$$

Equation of the hyper-surface

Metric of the ambient space $$dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 - dx_0^2 = ds^2$$

de-Sitter Local Coordinates

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 - x_0^2 = a^2$$

$$x_1^2 + x_2^2 + x_3^2 = a^2 + x_0^2 - x_4^2$$

$$x_1^2 + x_2^2 + x_3^2 = r^2$$

$$(t, r, \theta, \phi) \longleftarrow \quad x_0^2 - x_4^2 = r^2 - a^2$$

$$x_0 = \sqrt{r^2 - a^2} \cosh t$$

$$x_4 = \sqrt{r^2 - a^2} \sinh t$$

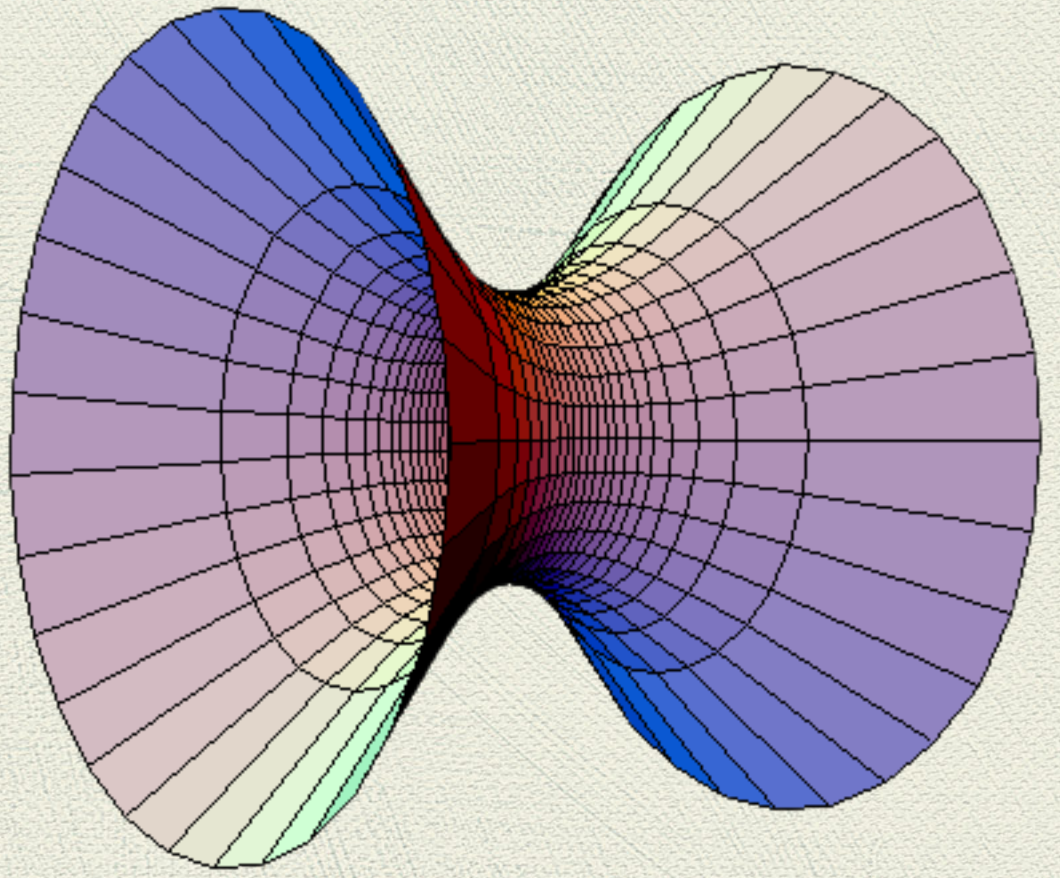
de-Sitter Metric

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 - dx_0^2$$

$$ds^2 = -(1 - \frac{r^2}{a^2})dt^2 + (1 - \frac{r^2}{a^2})^{-1}dr^2 + r^2 d\Omega$$

A-Constant Zero curvature in 4-dimension

Anti de Sitter Spacetime AdS^4



$$x_1^2 + x_2^2 + x_3^2 - x_4^2 - x_0^2 = -a^2$$

Equation of the hyper-surface

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - dx_4^2 - dx_0^2$$

Metric of the ambient space

Anti de Sitter local coordinates

$$x_1^2 + x_2^2 + x_3^2 - x_4^2 - x_0^2 = -a^2$$

$$x_0^2 + x_4^2 - (x_1^2 + x_2^2 + x_3^2) = a^2$$

$$x_0^2 + x_4^2 = a^2 \cosh^2 \tau$$

$(\tau, \rho, \theta, \phi)$ ←

$$x_1^2 + x_2^2 + x_3^2 = a^2 \sinh^2 \tau$$

$$x_0 = a \cosh \rho \cos \tau$$

$$x_4 = a \cosh \rho \sin \tau$$

Anti-de-Sitter Metric

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 - dx_0^2$$

$$ds^2 = a^2 \sinh^2 \tau (d\Omega^2 - d\rho^2) - a^2 \cosh^2 \tau d\tau^2$$

Duality in Physics

What is the meaning of Duality?

T and T* are two different theories,

g and G are coupling constants,

X and X* are different observables.

$$\langle X \rangle_T = \langle X^* \rangle_{T^*}$$

$$T(g) \leftrightarrow T^*(G)$$

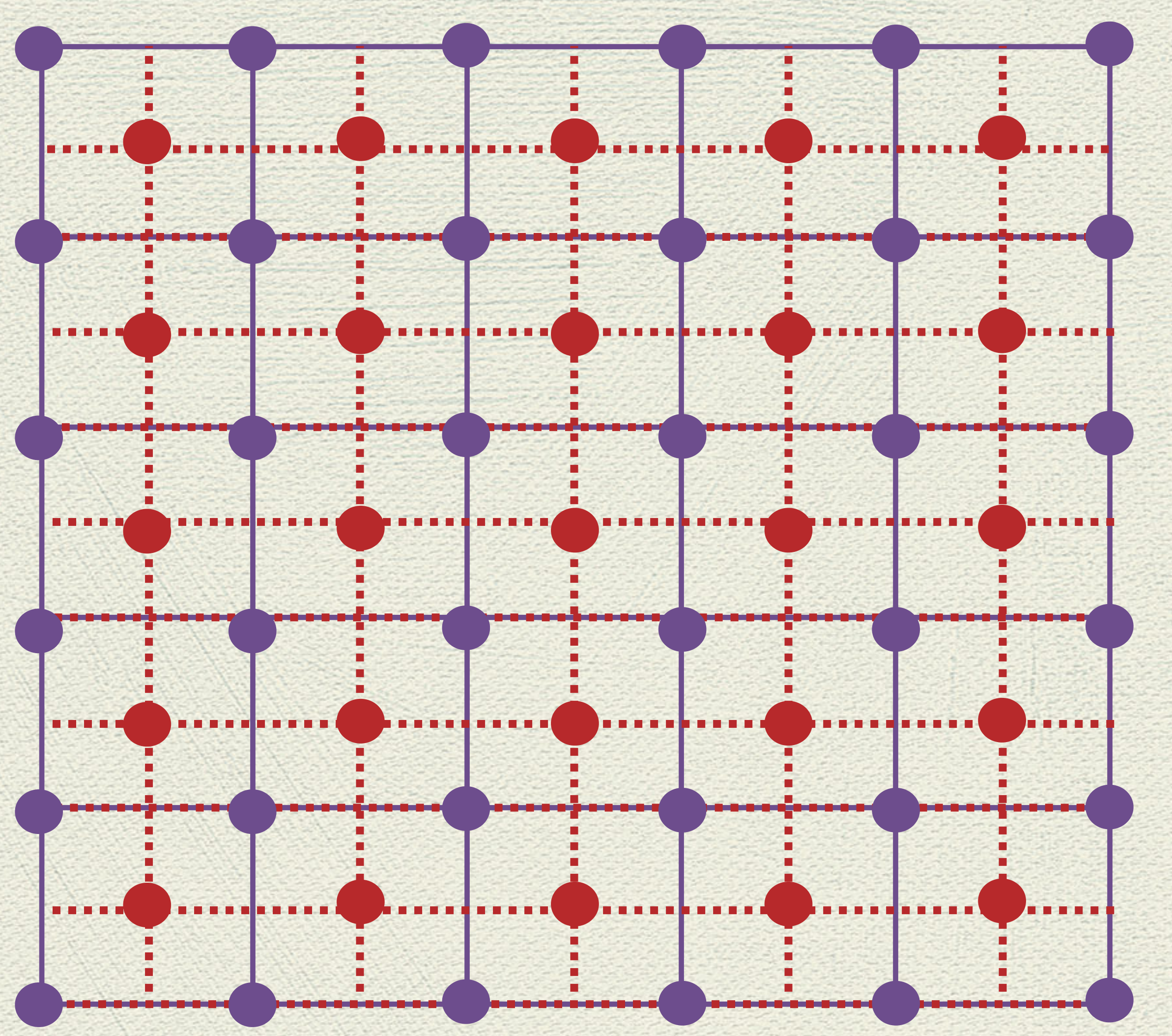
$$T(G) \leftrightarrow T^*(g)$$

An example: Duality between two classical systems

$$Z = \sum_{\{s_1, s_2, \cdots s_N\}} e^{-g \sum_{\langle i,j \rangle} s_i s_j}$$

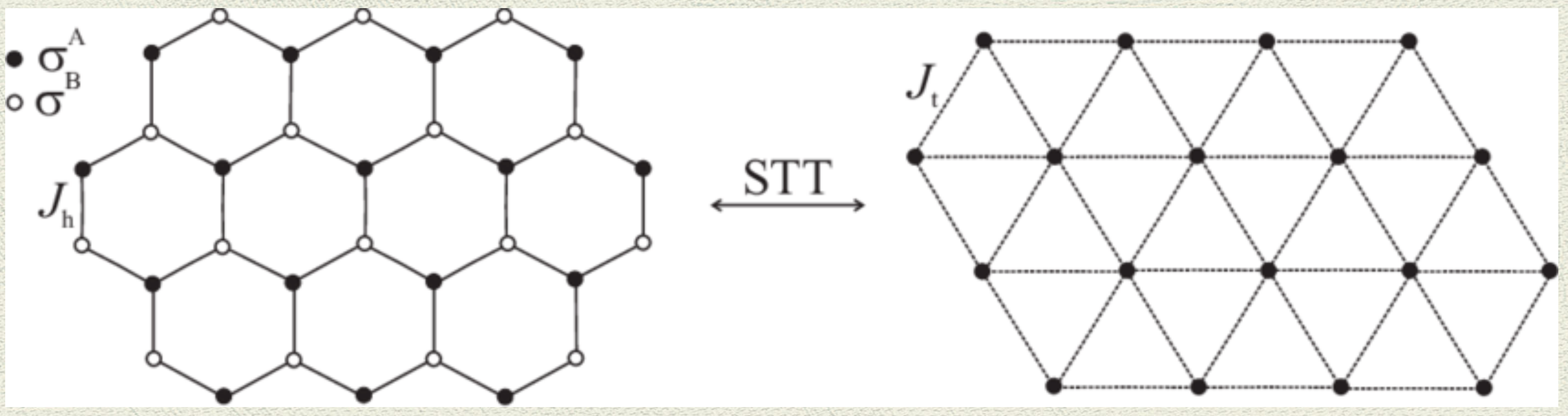
$$Z_{\square}(g) \quad \sim \quad Z_{\square}(G)$$

$$\frac{Z_{\square}(g)}{2^N (\cosh g)^{2N}} = \frac{Z_{\square}(G)}{e^{2GN}}$$



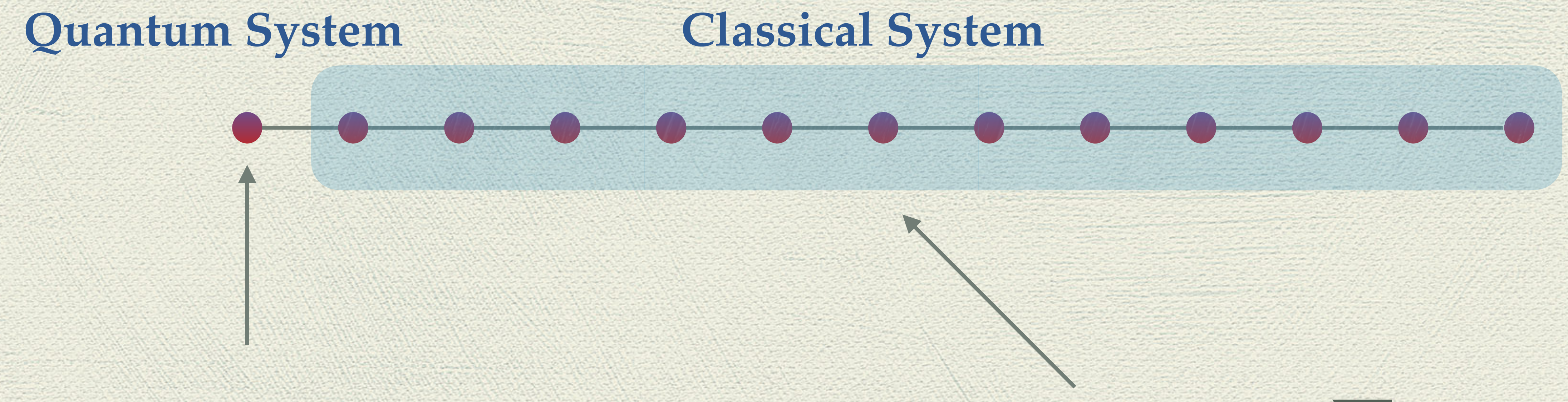
$$\sinh 2g \sinh(2G) = 1$$

Another example: Duality in Ising Model



$$Z_{\text{Triangular}}(g) \quad \sim \quad Z_{Hexagonal}(G)$$

An example: Duality between a classical and a quantum system



$$Z = \langle 0 | U^N | 0 \rangle$$

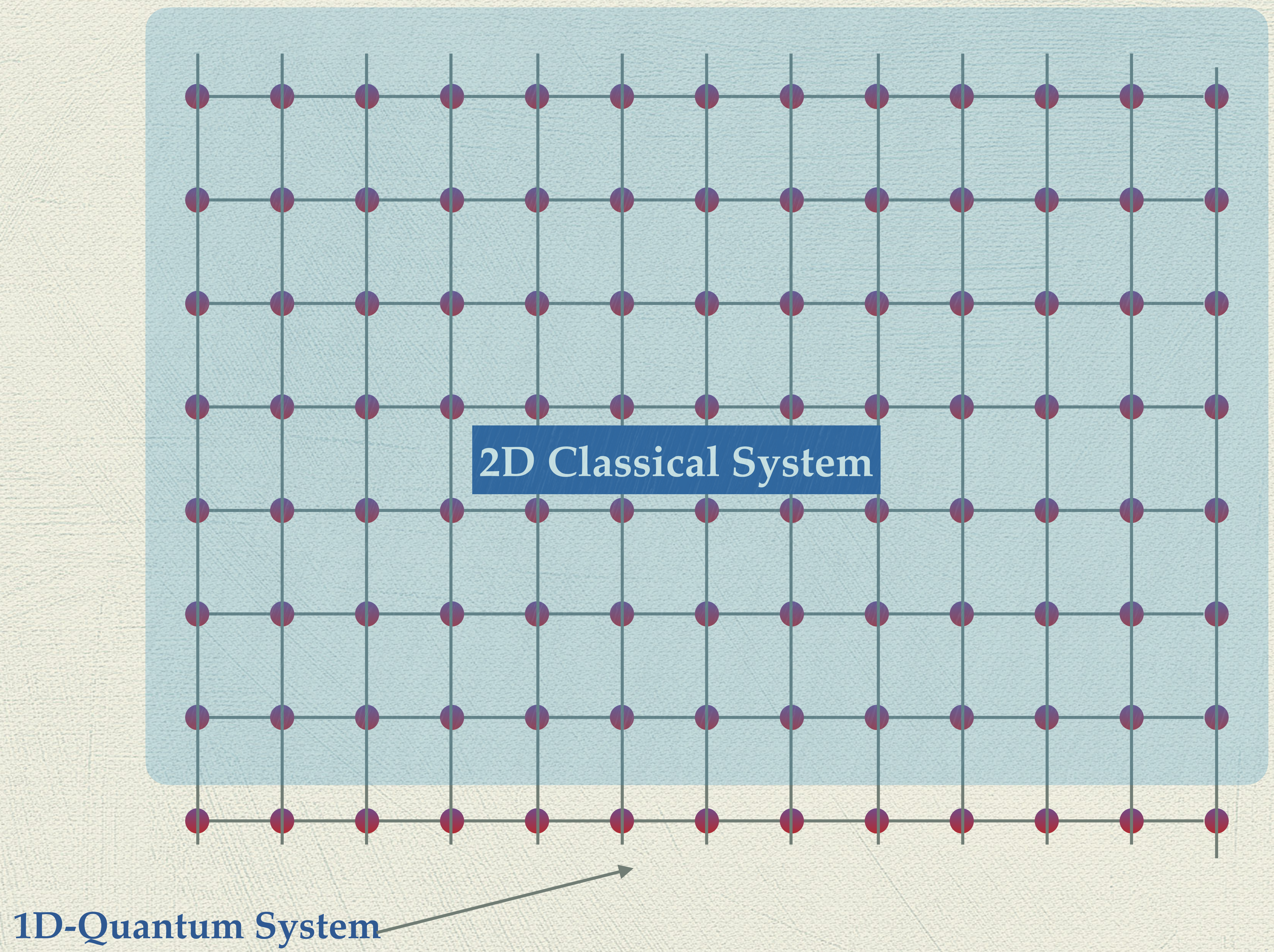
$$H = -J \sum_i s_i s_{i+1}$$

$$U = e^{-i\hat{H}}$$

$$Z = \sum_{s_1, \cdots s_N} e^{-\beta H}$$

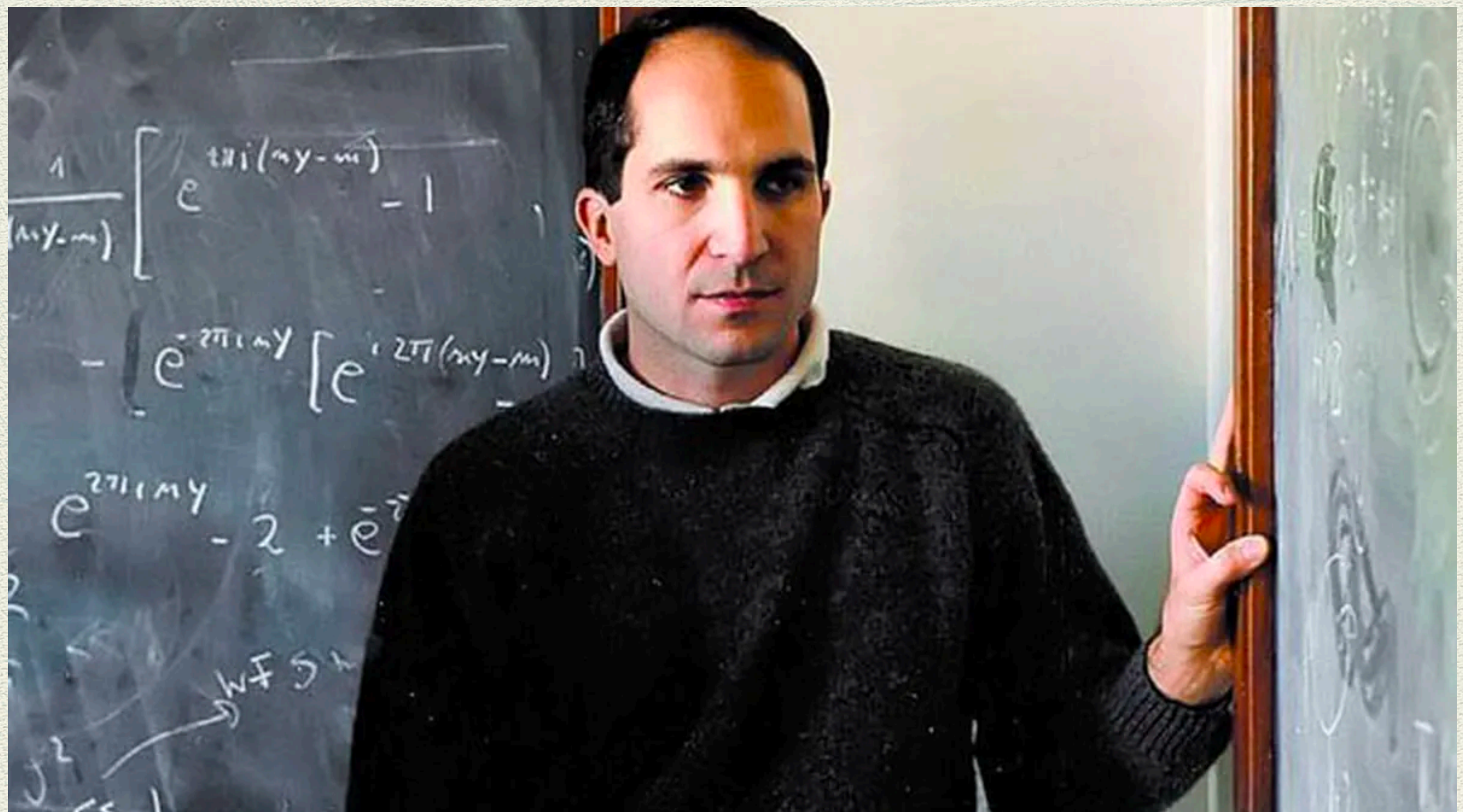
$$\langle s_i s_j \rangle = \langle 0 | \hat{\sigma}(i) \hat{\sigma}(j) | 0 \rangle$$

$$\langle s_i s_j \rangle = \frac{1}{Z} \sum_{s_1, \cdots s_N} s_i s_j \; e^{-\beta H}$$


2D Classical System
1D-Quantum System

Quantum System at the Boundary	Classical System in the Bulk
Time	Distance
Vaccum State	Thermodynamic Limit
Vaccum to Vaccum Amplitude	Partition Function
Heisenberg Operator $\hat{\sigma}(i)$	S_i
Correlation Function	Correlation Function
Energy Gap	Correlation Length

AdS-CFT duality



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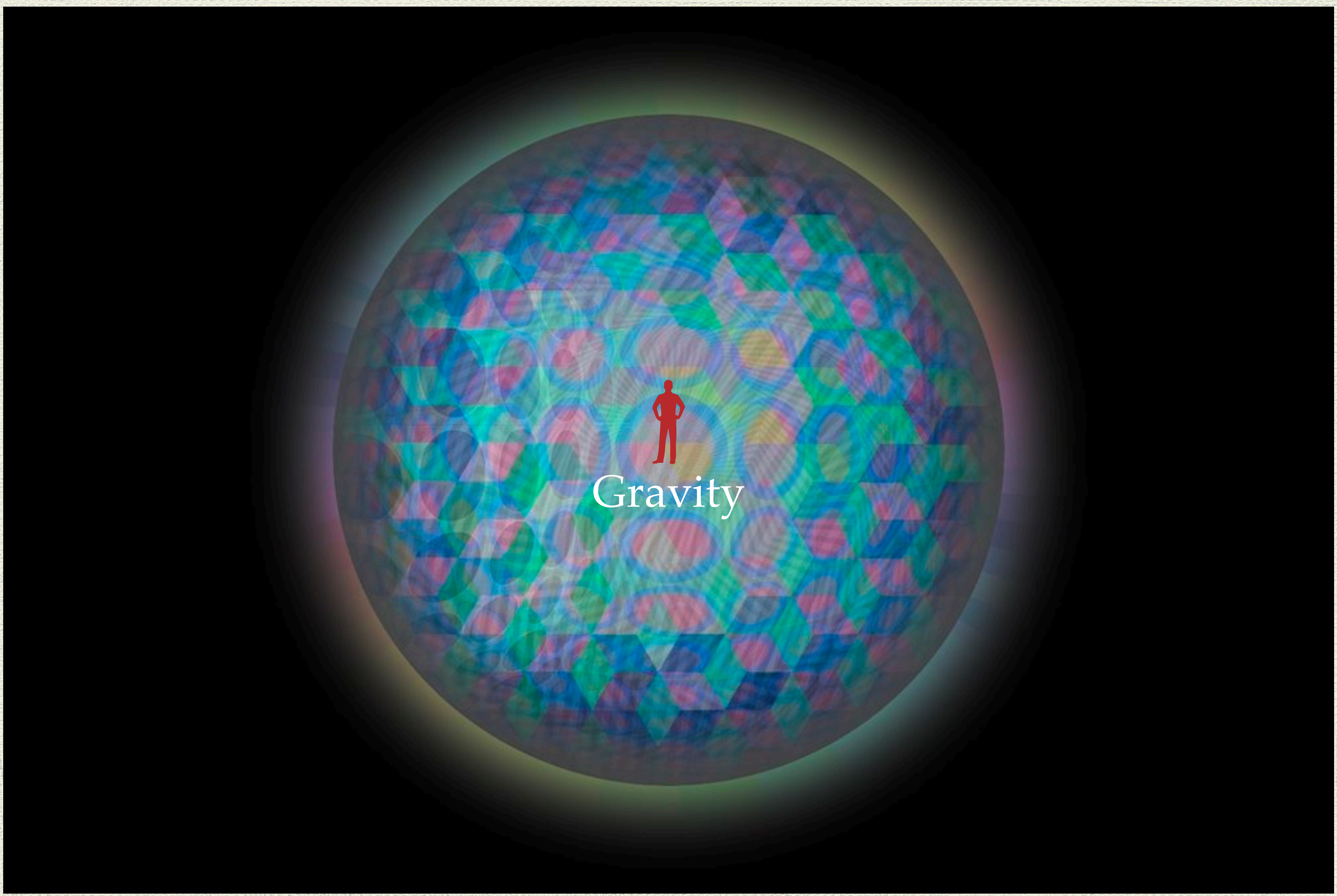
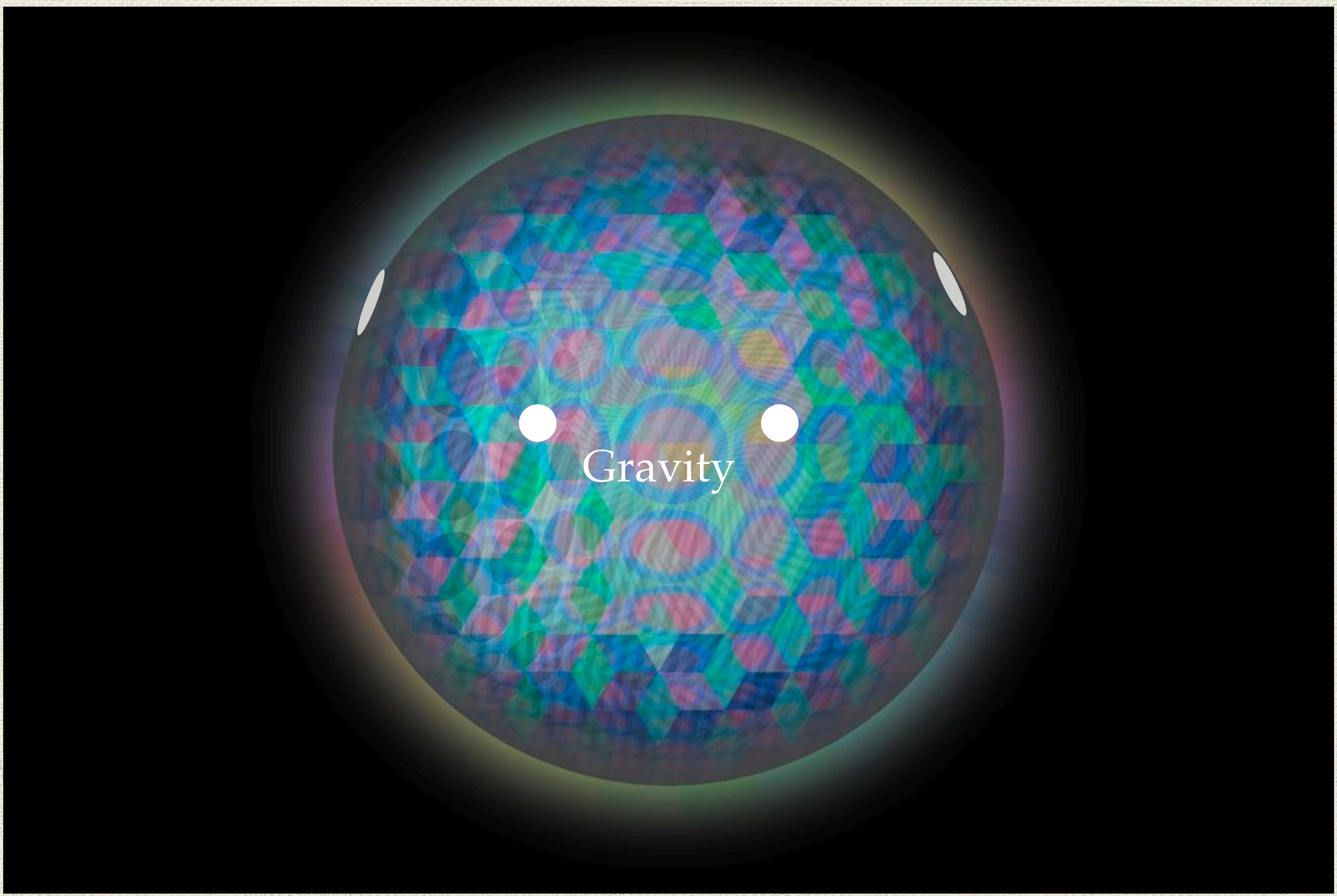
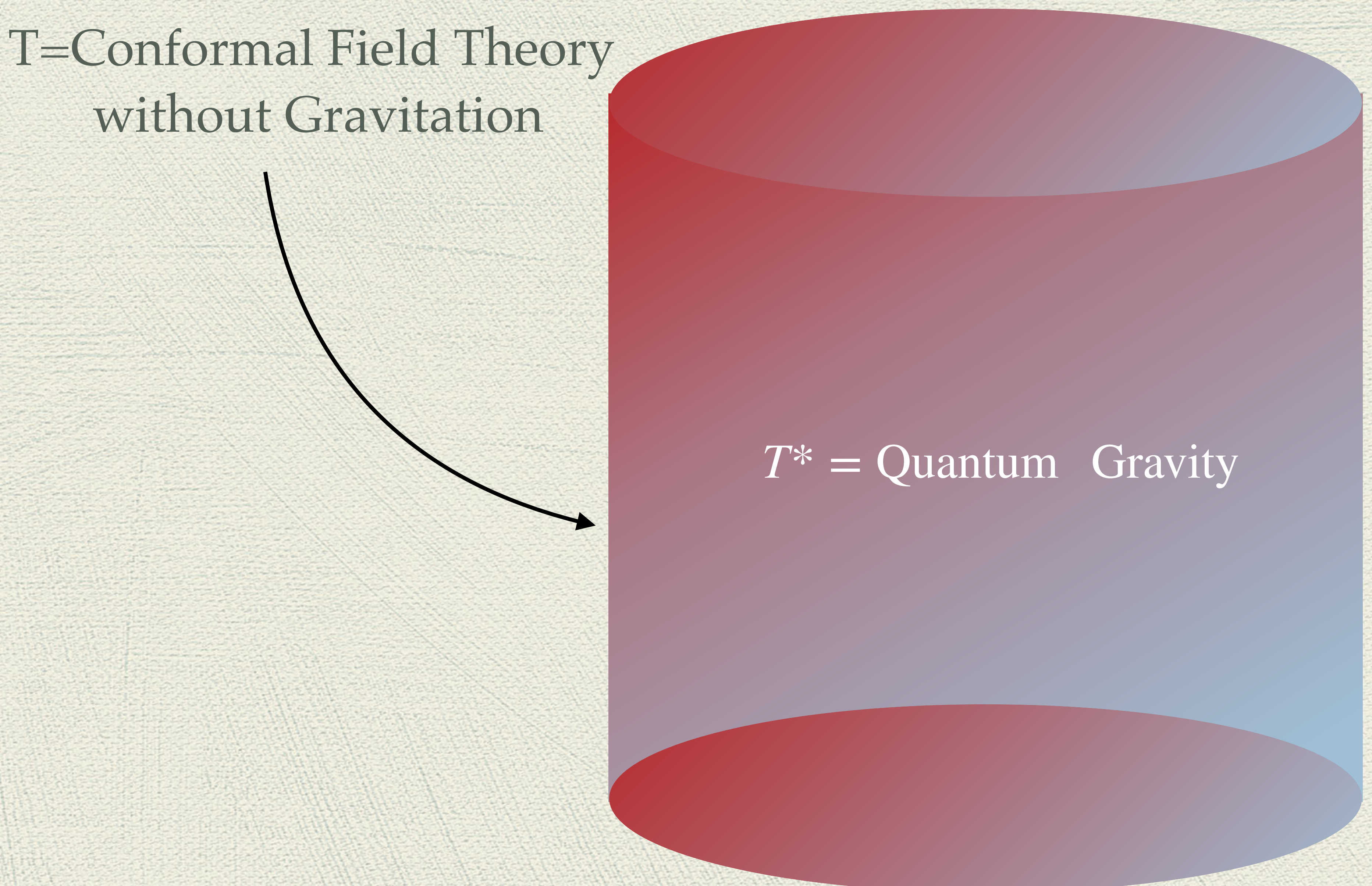
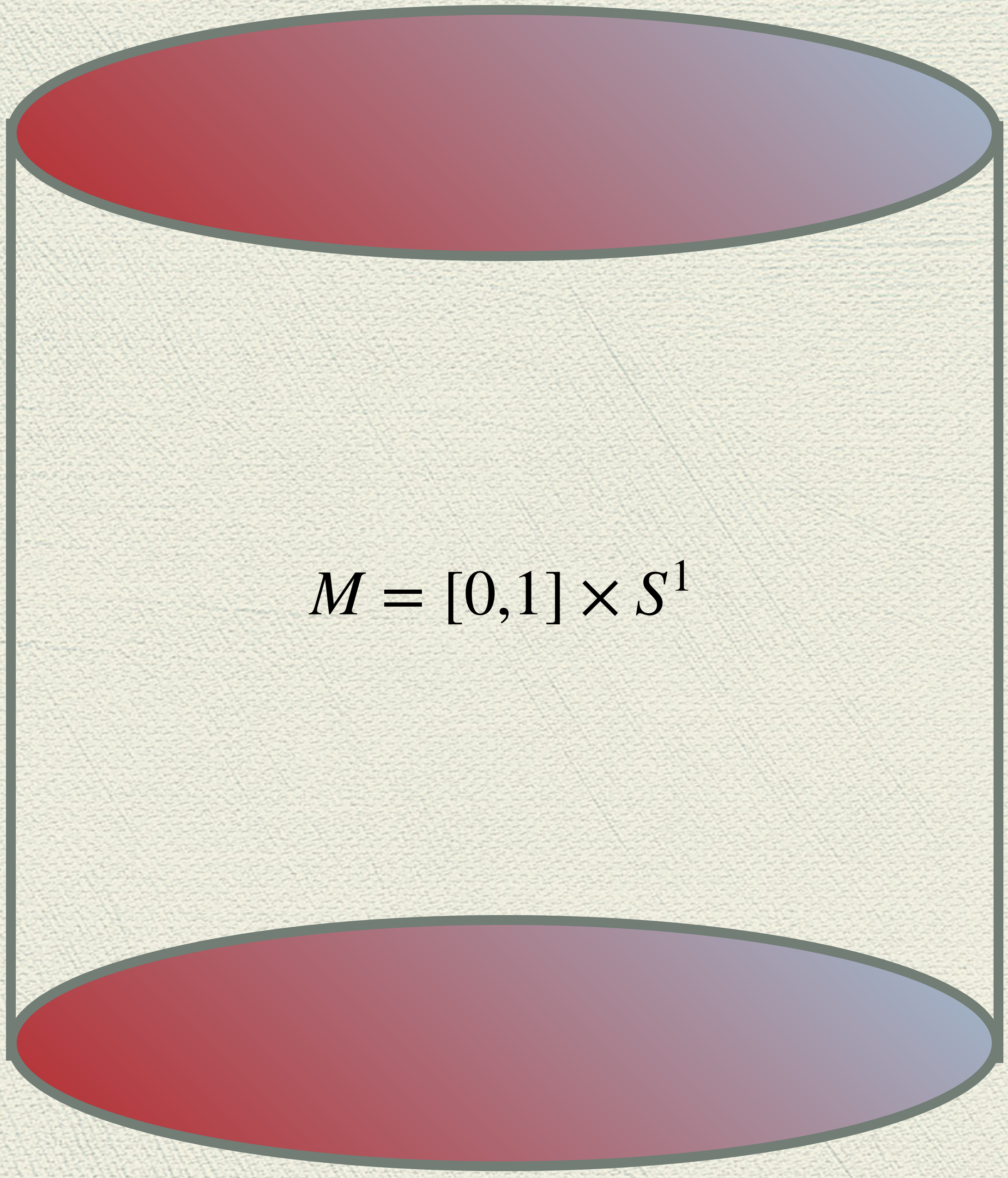

Gravity


Gravity

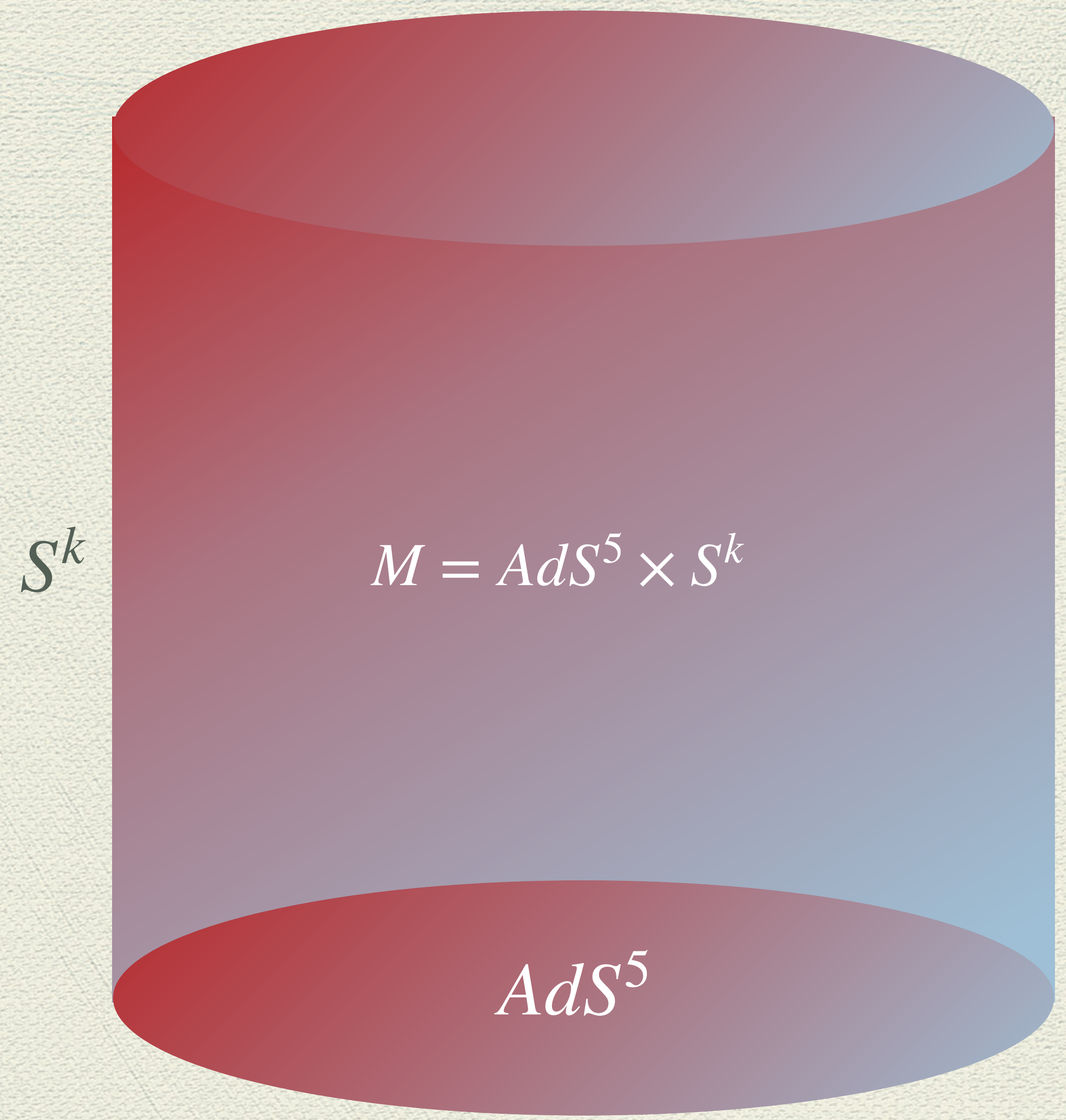
AdS/CFT Duality



$\partial M = \partial[0,1] \times S^1 \quad = \{0\} \times S^1 \;\cup\; \{1\} \times S^1$



$$\partial M = \partial(AdS^5) \times S^k$$



dim of $\mathrm{M} = 5 + k$

dim of $\partial\mathrm{M} = 4$

End of part II